Chapter 8
Three-Dimensional Viewing Operations

- Projections

Figure 8.1 Classification of planar geometric projections

Figure 8.2 Planar projection
Figure 8.6 Perspective projection

Figure 8.7 Multiview orthographic projection representation

Figure 8.8 Orthographic projection of a point on the xy plane
Example 8.1
Given an object as shown in Figure 8.9, find its front and side orthographic projections in the directions indicated by the arrows.

Solution
(a) The indicated front view is a projection on the $xy$ plane.

Based on Eq. 8.1, the matrix of points becomes

$$
\begin{bmatrix}
0 & 2 & 0 & 1 \\
2 & 2 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 2 & 0 & 1 \\
2 & 2 & 0 & 1 \\
2 & 1 & 0 & 1 \\
2 & 1 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1
\end{bmatrix}
$$

The projection is as shown in Figure 8.10a.
(b) The side view is obtained through an orthographic projection onto the yz plane.

Based on Eq. 8.2, the matrix of points becomes

\[
\begin{bmatrix}
0 & 2 & 0 & 1 \\
0 & 2 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 2 & 1 & 1 \\
0 & 2 & 1 & 1 \\
0 & 1 & 1 & 1 \\
0 & 1 & 2 & 1 \\
0 & 0 & 2 & 1 \\
0 & 0 & 2 & 1 \\
0 & 1 & 2 & 1 \\
0 & 1 & 1 & 1 \\
\end{bmatrix}
\]

The projection is as shown in Figure 8.10b.

Orthographic projections can be used to solve other types of problems. For example, in Figure 8.11, it may be necessary to have the inclined surface projected on the xy plane in its exact shape, without distortion.
Axonometric Projections (the most common used in engineering)

Isometric Projection: All three axes are equally foreshortened when projected.
\[
[M_{\text{TLT}}] = [T_R]_y^0 [T_R]_x^0
\]

\[
= \begin{bmatrix}
\cos \theta_y & 0 & -\sin \theta_y & 0 \\
0 & 1 & 0 & 0 \\
\sin \theta_y & \cos \theta_y & 0 & -\sin \theta_x \cos \theta_y \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta_x & \sin \theta_x & 0 \\
0 & \sin \theta_x & \cos \theta_x & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(8.3)

\[
= \begin{bmatrix}
\cos \theta_y & \sin \theta_y & \sin \theta_x & -\sin \theta_y \cos \theta_x & 0 \\
0 & \cos \theta_x & \sin \theta_x & 0 & 0 \\
\sin \theta_y & -\sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(8.4)

To finalize the isometric view, an orthographic projection onto the \(xy\) plane is obtained by:

\[
[M_{\text{ISO}}] = [M_{\text{TLT}}]
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
\cos \theta_y & \sin \theta_y & \sin \theta_x & 0 & 0 \\
0 & \cos \theta_x & \sin \theta_x & 0 & 0 \\
\sin \theta_y & -\sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(8.5)

\[
x^* = [1 \ 0 \ 0 \ 1] [M_{\text{ISO}}] = [\cos \theta_y \ \sin \theta_y \ \sin \theta_x \ 0 \ 1]
\]

\[
y^* = [0 \ 1 \ 0 \ 1] [M_{\text{ISO}}] = [0 \ \cos \theta_x \ 0 \ 1]
\]

\[
z^* = [0 \ 0 \ 1 \ 1] [M_{\text{ISO}}] = [\sin \theta_y \ -\sin \theta_x \cos \theta_y \ 0 \ 1]
\]

\[
|x^*| = \sqrt{\cos^2 \theta_y + \sin^2 \theta_y \sin^2 \theta_x}
\]

\[
|y^*| = \sqrt{\cos^2 \theta_x}
\]

\[
|z^*| = \sqrt{\sin^2 \theta_y + \sin^2 \theta_x \cos^2 \theta_y}
\]
1. \[ |x^*| = |y^*| \]
\[ \cos^2 \theta_y + \sin^2 \theta_y \sin^2 \theta_x = \cos^2 \theta_x \]
\[ 1 - \sin^2 \theta_y + \sin^2 \theta_y \sin^2 \theta_x = 1 - \sin^2 \theta_x \]
\[ \sin^2 \theta_y (\sin^2 \theta_x - 1) = -\sin^2 \theta_x \]
\[ \sin^2 \theta_y = \frac{\sin^2 \theta_x}{1 - \sin^2 \theta_x} \]

2. \[ |z^*| = |y^*| \]
\[ \sin^2 \theta_y + \sin^2 \theta_x \cos^2 \theta_y = \cos^2 \theta_x \]
\[ \sin^2 \theta_y + \sin^2 \theta_x (1 - \sin^2 \theta_y) = 1 - \sin^2 \theta_x \]
\[ \sin^2 \theta_y = \frac{1 - 2 \sin^2 \theta_x}{1 - \sin^2 \theta_x} \]

So that
\[ \frac{\sin^2 \theta_x}{1 - \sin^2 \theta_x} = \frac{1 - 2 \sin^2 \theta_x}{1 - \sin^2 \theta_x} \]

And
\[ \sin^2 \theta_x = 1 - 2 \sin^2 \theta_x \]
\[ \sin^2 \theta_x = \frac{1}{3} \]
\[ \theta_x = \pm 35.26^\circ \]

Substitution of the value of \( \theta_x \) into Eqs. 8.13 or 8.15 yields
\[ \sin^2 \theta_y = \frac{1}{2} \]
\[ \theta_y = \pm 45^\circ \]

\[ \tan A = \frac{x_y^*}{x_x^*} = \frac{\sin \theta_y \sin \theta_x}{\cos \theta_y} \]

Since
\[ \theta_y = 45^\circ, \sin \theta_y = \cos \theta_y, \]
and
\[ \tan A = \pm \sin \theta_x = \pm \sin (35.26)^\circ \]
so that
\[ A = \pm 30^\circ \]
The forshorten factor:

\[ F = \frac{|y'|}{1} = \sqrt{\cos^2 \theta_x} = \sqrt{\frac{2}{3}} = 0.8165 \]

Example 8.2
Determine the isometric projection of the block described in Figure 8.9, for \( \theta = 45^\circ \), and \( \theta_x = 35.26^\circ \).

[Solution]
The solution is obtained with Eq. 8.5.

\[
\begin{bmatrix}
\cos \theta_y & \sin \theta_y & \sin \theta_x & 0 & 0 \\
0 & \cos \theta_x & 0 & 0 \\
\sin \theta_y & -\sin \theta_x & \cos \theta_y & 0 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]

For the given values of \( \theta_x \) and \( \theta_y \), this becomes:
.. **Trimetric Projection**: The transformation matrix causes pure rotation. Thus, the coordinate axes remain orthogonal when projected.

.. **Dimentric Projection**: Two of the three axes are equally foreshortened when projected. The length of receding axis is given a specific foreshorten value.

**Example**: After projection, two unit vectors of x and y axes are equally foreshorten, and the length of unit vector of z axis becomes half (or 0<foreshorten factor<1), therefore, we gain $\theta_x = 20.705$, $\theta_y = 22.208$. 

\[
\begin{bmatrix}
0 & 2 & 0 & 1 \\
2 & 2 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 2 & 1 & 1 \\
2 & 2 & 1 & 1 \\
2 & 1 & 1 & 1 \\
2 & 1 & 2 & 1 \\
2 & 0 & 2 & 1 \\
0 & 0 & 2 & 1 \\
0 & 1 & 2 & 1 \\
0 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
0.707 & 0.408 & 0 & 0 \\
0 & 0.816 & 0 & 0 \\
0.707 & -0.408 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0.0 & 1.632 & 0.0 & 1.0 \\
1.414 & 2.448 & 0.0 & 1.0 \\
1.414 & 0.816 & 0.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 1.0 \\
0.707 & 1.224 & 0.0 & 1.0 \\
2.121 & 2.040 & 0.0 & 1.0 \\
2.12 & 1.224 & 0.0 & 1.0 \\
2.828 & 0.816 & 0.0 & 1.0 \\
2.828 & 0.0 & 0.0 & 1.0 \\
1.414 & -0.816 & 0.0 & 1.0 \\
1.414 & 0.0 & 0.0 & 1.0 \\
0.707 & 0.408 & 0.0 & 1.0
\end{bmatrix}
\]
• Oblique Projection (Shearing transformation matrix)
The object is customarily positioned so that one of its principal faces is parallel to the plane of projection. This principal face appears in its true size and shape on the plane of projection. The three main axes of an oblique projection appear such that two of them are orthogonal in the plane of the paper, and the receding axis is oriented at any convenient angle (usually from 15° to 45° for proper representation of the object).

Fig. 5.6 Oblique drawing: contoured front plane

Fig. 5.7 Oblique circles

• Transformations – $l$ gives the foreshortening ratio of any line perpendicular to the $z = 0$ plane, after projection. $\theta$ is the angle between the projection and the horizontal axis.

$$x^* = l \cos \theta$$

$$y^* = l \sin \theta$$

and the oblique projection matrix becomes

$$[M_{\text{Obl.}}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ l \cos \theta & l \sin \theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
1. Shearing of the object in space in a direction parallel to the plane of projection.
2. Orthographic projection onto the plane of projection.

- Special cases of oblique: Cavalier and Cabinet projection
  - Cavalier Projection: Two axes appear perpendicular and are not foreshortened; the third axis is inclined with respect to the horizontal and is not foreshortened ($l = 1$). It is the result of projecting rays from the object to the picture plane at a 45° angle to the plane.
  - Cabinet Projection: Same as Cavalier projection, only the third axis is foreshortened by a factor of 1/2 ($l = 1/2$).
    
    The most commonly used value for $\theta$ in Cabinet projection are 30° to 45°.

Oblique projections have been useful in representing objects with curved features that can be placed on the plane of projection parallel to the observer.
Example 8.3
A tetrahedron is defined by the coordinates of its vertices, as
\[ P_1(3, 4, 0), P_2(1, 0, 4), P_3(2, 0, 5), P_4(4, 0, 3) \]

Find the oblique cavalier projection onto a viewing surface in the \( z = 0 \) plane. The angle between the \( z \) axis and the horizontal should be \( 45^\circ \) in the projection.

[Solution]
The oblique matrix is given in Eq. 8.23. For cavalier projections, \( l = 1 \), and the matrix, with \( \theta = 45^\circ \), becomes
\[
\begin{bmatrix}
3 & 4 & 0 & 1 \\
5/9 & 0 & 0 & 1 \\
5/2 & 0 & 0 & 1 \\
\end{bmatrix}
\]
The transformed coordinates of the original tetrahedron are
\[
\begin{bmatrix}
3.83 & 2.83 & 0 & 1 \\
5.54 & 3.54 & 0 & 1 \\
6.12 & 2.12 & 0 & 1 \\
\end{bmatrix}
\]

Perspective Projections
1-point perspective projection
The perspective projection of \( P(x, y, z) \) onto the \( xy \) plane, \( P^*(x^*, y^*, 0) \), with the center of projection a distance \( Z_{cp} \) along the \( Z \) axis.

Figure 8.19 Perspective projection of point \( P \) on the \( xy \) plane
1. Center of projection on the x-axis

2. Center of projection on the y-axis
3. Center of projection on the z-axis
   - 2-point perspective projection
   - 3-point perspective projection

For 2-point perspective:

\[
\begin{bmatrix}
1 & 0 & 0 & r \\
0 & 1 & 0 & s \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & s \\
0 & 0 & 1 & t \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

For 3-point perspective:

\[
\begin{bmatrix}
1 & 0 & 0 & r \\
0 & 1 & 0 & s \\
0 & 0 & 1 & t \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Example 8.4
Find the perspective projection of the tetrahedron given in Example 8.3 onto a projection plane at \( z = 0 \). The center of projection should be located at \( z_{cp} = -5 \).

[Solution]
Equation 8.28 is used in the solution of this problem. The projected points are

\[
\begin{bmatrix}
P' \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \begin{bmatrix} M_{PER} \end{bmatrix} = \begin{bmatrix}
3 & 4 & 0 & 1 \\
1 & 0 & 4 & 1 \\
2 & 0 & 5 & 1 \\
4 & 0 & 3 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & \frac{1}{5} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
P' \end{bmatrix} = \begin{bmatrix}
3 & 4 & 0 & 1 \\
1 & 0 & 0 & 1.8 \\
2 & 0 & 0 & 2 \\
4 & 0 & 0 & 1.6
\end{bmatrix} = \begin{bmatrix}
3 & 4 & 0 & 1 \\
5 & 9 & 0 & 1 \\
1 & 0 & 0 & 1 \\
5 & 2 & 0 & 1
\end{bmatrix}
\]

Figure 8.22 shows the result of this projection.
Vanishing Points

The perspective transformation of a point located at infinity on the z axis can be represented as \([0 \ 0 \ 1 \ 0]\). An infinite point can be represented as \([x \ y \ z \ 0]\). Applying the perspective transformation to this point yields

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & t \\
0 & 0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 1 & t \\
\end{bmatrix}
\]

Vanishing point at \([0 \ 0 \ 1/t \ 1]\)

\[
\begin{bmatrix}
1 & 0 & 0 & r \\
0 & 1 & 0 & s \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The two vanishing points are \([1/r \ 0 \ 0 \ 1], [0 \ 1/s \ 0 \ 1]\).

\[
\begin{bmatrix}
1 & 0 & 0 & r \\
0 & 1 & 0 & s \\
0 & 0 & 1 & t \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
• Special Techniques

Two-point perspective projection

\[
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta & 0 \\
0 & 1 & 0 & 0 \\
\sin \theta & 0 & \cos \theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & m & n & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & 0 & \sin \theta \\
0 & 1 & 0 & \frac{n}{z_{cp}} \\
\sin \theta & 0 & 0 & \frac{-z_{cp}}{z_{cp}} \\
0 & m & 0 & \left(1 - \frac{n}{z_{cp}}\right)
\end{bmatrix}
\]

The cube is rotated by $\theta$ about the y axis and translated by [0 m n], before a 1-point perspective projection onto the $z = 0$ plane, with center of projection along the z axis.

\[
[T_R]_{y} \cdot [T_{TR}^{(0,m,n)}][M_{PER}]
\]
Three-point perspective projection
The cube is first rotated about the y axis and then about the x axis, before applying the same perspective projection.

\[
\begin{bmatrix}
T_{R_y}^\theta & T_{R_x}^\Phi & M_{PER}
\end{bmatrix}
\]

Example 8.5
For the unit cube shown in Figure 8.24, determine the 2-point perspective projection obtained by rotating the cube 30° about the y axis and translating it by (0, 3, -3). The center of projection is at (0, 0, 2)

[Solution]
Two-point perspective projection is applied to the point matrix of the cube, as follows:

\[
P^* = \begin{bmatrix}
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
0.866 & 0 & 0.250 \\
0.866 & 4.0 & 2.75 \\
0.5 & 0 & -0.433 \\
0 & 3.0 & 2.5 \\
1.37 & 3.0 & 2.32 \\
1.37 & 4.0 & 2.32 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 3.0 & 0 & 2.5 \\
0.866 & 3.0 & 0 & 2.75 \\
0.866 & 4.0 & 0 & 2.75 \\
0 & 4.0 & 0 & 2.5 \\
0.5 & 4.0 & 0 & 2.07 \\
0.5 & 3.0 & 0 & 2.07 \\
1.37 & 3.0 & 0 & 2.32 \\
1.37 & 4.0 & 0 & 2.32 \\
\end{bmatrix}
\]
Two vanish points along x- and z-axis, they are (3.464, 0) and (-1.155, 0), respectively.
The viewing process for three-dimensional models is analogous to a camera taking pictures from different positions in space and in various orientations. To define the position of the camera in space, it is necessary to establish a new
The Viewing Reference Coordinate (VRC) system, which will be centered at the window.

Three parameters are necessary to define the VRC.  
1. the View Reference Point (VRP) defined in world coordinates as a point in the direction of which the camera is looking.  
2. the View Plane Normal (VPN) in the normal direction to the view plane, defining its orientation.  
3. the “up” direction for the camera. The View-Up Vector (VUP, vector $\mathbf{v}$).

Figure 8.27 Camera analogy to the viewing process

Figure 8.28 Camera viewfinder corresponds to the window
Figure 8.29 Establishing the Viewing Reference Coordinate (VRC) system

Window in VRC is (0,0) to (1,1) — Normalized window

Figure 8.30 Effect on the image of a change in the direction of the VUP
To create a view of an object by user
PHIGS uses a set of default specifications to represent the viewing parameters as follows:

VRP in WC  (0, 0, 0)
VPN in WC  (0, 0, 1)
VUP in WC  (0, 1, 0)
Window in VRC (0, 1, 0, 1)

A sequence of transformations are list below.
1. Translate the view reference point to the WC origin.
2. Rotate about the $x_w$ axis so that $N$ falls on the $x_wz_w$ plane.

3. Rotate about the $y_w$ axis, so that $N$ coincides with $z_w$.

4. Rotate about $z_w$ so that all axes coincide.

$$T = \begin{bmatrix} T_{TR} \\ T_x \end{bmatrix} \begin{bmatrix} T_R \end{bmatrix} \begin{bmatrix} T_y \end{bmatrix} \begin{bmatrix} T_z \end{bmatrix}$$
• View Volumes (Front/Side/Top Views creation)
The position of the window in the view plane and the type of projection (parallel or perspectives) determine a view volume.

Figure 8.32 view volumes for perspective and parallel projections

• For parallel projection, the Projection Reference Point (PRP) determines the direction of projection. For perspective projections, the PRP defines the center of projection. Figure 8.31

• Yon (back) plane and Hither (front) plane are used to define a finite section of the view volume.

Figure 8.33 Front (hither) and back (yon) planes defining a finite section of the view volumes

• The appropriate viewing parameters (VRP, VPN, VUP, PRP, Window) should be chosen to produce the desired results.
Example 8.6
Consider the chamfered block in Figure 8.34, described in world coordinates. Establish the viewing parameters (PHIGS) needed to produce
(a) An orthographic front view
(b) An orthographic top view

[Solution]
The viewing parameter values that will produce the desired projections are not unique. Various possible combinations can be found. The important thing to remember is the coordinate system in which the parameter is established, WC or VRC.

(a) *Front View.* Figure 8.35 gives one solution to this problem. Notice that the PRP, in the VRC, establishes the direction of viewing as it connects to the center of the window. As seen in Figure 8.35, this line passes...
through the approximate center of the front face and is parallel to the z axis.

(b) **Top View.** Figure 8.36 shows one possible solution. Notice the rotation of the UVN system with respect to the WC system. The PRP is set to (10, 5, 25) in this new orientation of the UVN.

### Front View

<table>
<thead>
<tr>
<th>PHIGS Viewing Parameters</th>
<th>Value</th>
<th>Coordinate System</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRP</td>
<td>(0,0,0)</td>
<td>WC</td>
</tr>
<tr>
<td>VPN</td>
<td>(0,0,1)</td>
<td>WC</td>
</tr>
<tr>
<td>VUP</td>
<td>(0,1,0)</td>
<td>WC</td>
</tr>
<tr>
<td>PRP</td>
<td>(5,5,75)</td>
<td>VRC</td>
</tr>
<tr>
<td>Window</td>
<td>(-1,11,-1,11)</td>
<td>VRC</td>
</tr>
<tr>
<td>Project</td>
<td>Parallel</td>
<td></td>
</tr>
</tbody>
</table>

### Top View

<table>
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<th>PHIGS Viewing Parameters</th>
<th>Value</th>
<th>Coordinate System</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRP</td>
<td>(10,0,30)</td>
<td>WC</td>
</tr>
<tr>
<td>VPN</td>
<td>(0,1,0)</td>
<td>WC</td>
</tr>
<tr>
<td>VUP</td>
<td>(-1,0,0)</td>
<td>WC</td>
</tr>
<tr>
<td>PRP</td>
<td>(10,5,25)</td>
<td>VRC</td>
</tr>
<tr>
<td>Window</td>
<td>(-1,21,-5,15)</td>
<td>VRC</td>
</tr>
<tr>
<td>Project</td>
<td>Parallel</td>
<td></td>
</tr>
</tbody>
</table>
• Clipping in three dimensions
  If the view volume is converted to a unit cube defined by
  \[ x=0, \ x=1, \ y=0, \ y=1, \ z=0, \ z=1 \]
  Then this unit cube is referred to as the \textit{normalized} or \textit{canonical view volume}.
• Three types of canonical view volumes are
  1. Orthographic projection view volume – as a rectangular parallelepiped
  2. Oblique projection view volume – shear to align to view plane normal with the projectors

Figure 8.36 Establishing the canonical view volume for an orthographic projection
3. perspective projection view volume – shear in the x and y direction to locate the center of projection on the normal to the window or view plane (Figure 8.38), then truncate the pyramid volume into a parallelepiped volume (Figure 8.39)
Once the canonical view volume is obtained, clipping (Cohen-Sutherland algorithm) is performed against its faces. Intersection calculations are invoked then.

Figure 8.39 Scaling transformation needed to change truncated pyramid into a parallelepiped

Figure 8.40 Six-bit regions used for three-dimensional clipping