Chapter 12 Solid Modeling

・ Wireframe, surface, solid modeling

Solid modeling gives a complete and unambiguous definition of an object, describing not only the shape of the boundaries but also the object's interior and exterior regions.

・ Disadvantages of wireframe representations

1. Ambiguous in the way to represent an object.
2. Not suitable for
   a. Mass property calculations
   b. Hidden surface removal
   c. Shaded images generation

・ Solid Representation

・ Real world object satisfy specific properties causing them to be
1. **Bounded** – limited boundary, contain interior of the solid
2. **Homogeneously three-dimensional** – no dangling edge or faces, the boundary is always in contact with the interior of the solid.
3. **Finite** – finite in size and limited amount of information (area, mass and volume determinations).
Formal properties of geometric modeling
1. Domain or coverage – define object classes
2. Validity – legal model
3. Completeness – complete solid with enough data for geometric calculation performed
4. Uniqueness

Solid models representation schemes
1. CSG (Constructive Solid Geometry)
2. B-Rep (Boundary Representation)
3. Sweeping
4. Spatial Enumeration

Fundamental geometric principles
1. Geometry
2. Topology
3. Geometric closure
4. Set theory and operations
5. Set membership classification

Basics of Solid Modeling Theory
The fundamental geometric principles
1. **Geometry and topology** –
   **Definition:** Geometry relates to the information containing shape-defining parameters, such as the coordinates of the vertices in a polyhedral object.
   **Definition:** Topology describes the connectivity among the various geometric components, i.e. the relational information between the different parts of an object.
2. **Geometric closure** – Bounded, Finite, No dangling

3. **Set Theory** –
   A set is defined as any collection of objects, called “elements” or “members.”
   Universal set \( W \), containing all points in \( E^3 \) space, and the null set, \( \emptyset \), no elements.
   Set operations: union (\( \cup \)), intersection (\( \cap \)), difference (\( - \)).

4. **Regularized set operations** – Boolean operations ensure the validity of geometric models, avoiding the creation of nonsense objects.

5. **Set membership classification** – two sets \( X \) and \( S \), check how various parts of \( X \) can be assigned to \( S \) as being on its interior, exterior, or on its boundaries. \( X \) is partitioned into subsets \( X_{inS} \), \( X_{onS} \), \( X_{outS} \).
Constructive Solid Geometry (CSG)

A CSG model assumes that physical objects can be created by combining basic elementary shapes (primitives) through specific rules.

CSG primitives are represented by the intersection of a set of half-spaces, as shown in Figure 12.10.

Quadric surfaces are commonly used in CSG because they represent the most commonly used surface in mechanical design produced by the stand operations of milling, turning, rolling. E.g. planar surfaces are obtained through rolling and milling, cylindrical surface through turning, spherical surfaces through cutting done with a ball-end cutting tool.

Data structure for the CSG representation is based on the binary tree structure.
CSG example

Disadvantages:

1. The way of primitive combinations for the CSG representation is not unique. It has been found through the use of different primitives and Boolean operations.

2. The CSG doesn’t specify quantitative values for the new solid (unevaluated model). The new model must be checked through a boundary evaluation routine with will supply quantitative information about its vertices, edges, faces.

3. It shorts of intersection calculation in the form of curve/curve, curve/surface, or surface/surface intersections.
Boundary Representation – B-rep

The B-rep is built on the idea that a physical object is enclosed by a set of faces, which themselves belong to closed and orientable surfaces.

**Geometric and topology entities**

<table>
<thead>
<tr>
<th>Point</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curve</td>
<td>Edge</td>
</tr>
<tr>
<td>Surface</td>
<td>Face</td>
</tr>
</tbody>
</table>

The Eular-Poincaré law gives a quantitative relationship among faces, edges, vertices, faces’ inner loops, bodies or through holes (genus) in solids.

**The Eular-Poincaré law**

\[ F - E + V - L = 2(B - G) \]

A loop represents a connected portion of the boundary of a face. The face’s inner loop represents the connected portion of the boundary of two faces. Eular law is not only suit for solids with planar faces, but also for curved objects with closed curved faces or edges.

**Simplest form:** \[ F - E + V = 2 \]
• Faceted representation

**Advantages:** easy to add new surface types and a small amount of vary simple geometric data will satisfy all the needs.

• Sweep representation

Translation and rotational sweeping are used to create the sweep solid. In engineering applications sweeping can be used to detect possible interference between moving parts, or simulate and analyze material removal operations in manufacturing (tool moving along a predefined path intersects the raw stock of the part).
• Spatial enumeration schemes
The smaller the cell is, the more accurate the model is.
Spatial enumeration schemes have the advantage of easy access to any part of the model and the assurance of spatial uniqueness.
2D – quadtrees
3D – octrees
Solid modeling systems

Comparison between CSG and B-rep representations.

<table>
<thead>
<tr>
<th></th>
<th>Storage of Model</th>
<th>Detail Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSG</td>
<td>Implicit</td>
<td>Low</td>
</tr>
<tr>
<td>B-rep</td>
<td>Explicit</td>
<td>High</td>
</tr>
</tbody>
</table>

Advantages (A) and Disadvantages (D) comparisons.

<table>
<thead>
<tr>
<th></th>
<th>Complexity</th>
<th>Uniqueness</th>
<th>History of Construction</th>
<th>Use in Interactive Environment</th>
<th>Local Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSG</td>
<td>A</td>
<td>D</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>B-rep</td>
<td>D</td>
<td>A</td>
<td>D</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

Conversion among representations.

Solid modeling systems are characterized as “mostly CSG” or “mostly B-rep”. (Dual representation of a solid model)

Mostly CSG – model creation and editing is done only in the CSG form; once the model is created, a boundary evaluator algorithm is used to obtain a boundary representation (store internally along with the CSG tree).

Mostly B-rep – the user can create the model in either CSG or B-rep, but the CSG representation is discarded by the system.
• NURBS can exactly represent quadric surfaces, so internal operations in the modeler, such as the calculation of surface/surface intersection are accomplished with a single algorithm. (minimize the amount of geometric software required in the modeler)

• **Feature modeling**
  Features can link CAD and CAM in an efficient way.
  The feature modeler contains not only a geometric and topological structure but also support geometric characteristics of a part. (shapes of holes, cutouts, slots, chamfers, ribs, etc.)

Three fundamental approaches to feature modeling
1. Human-assisted feature recognition
   E.g. Tolerance or surface of model are created and stored in the database and later used by process planning systems.
2. Automatic feature recognition
   Find and extract form features the correspond to some predefined geometric pattern. (very difficult)
3. Design by feature

• **Applications of Solid Modeling to Engineering**
  Solid modeling is commonly used in engineering to aid visual analysis of a design idea, mass property calculations, and static interference analysis.
A simple Solid Modeling System (sample)
Four steps for developing the solid modeling system
1. CSG representation for user input (create quadric primitives)
2. Conversion to a faceted representation
3. Intersection calculation – Boolean operations
4. Rendering